

MSLC Workshop Series

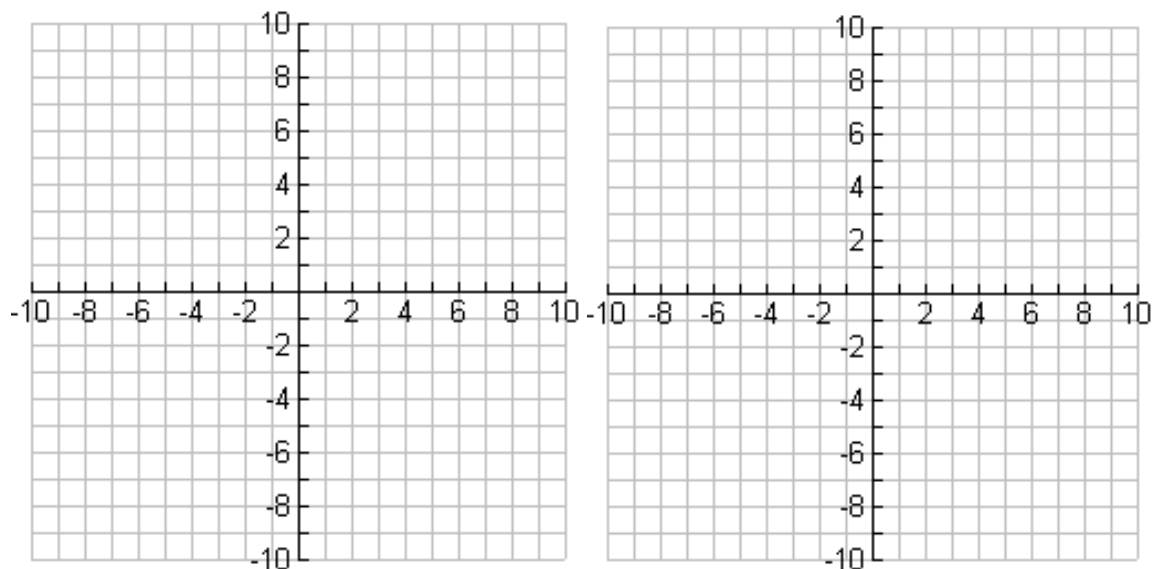
Math 1151 – Workshop 1

Limits and Continuity

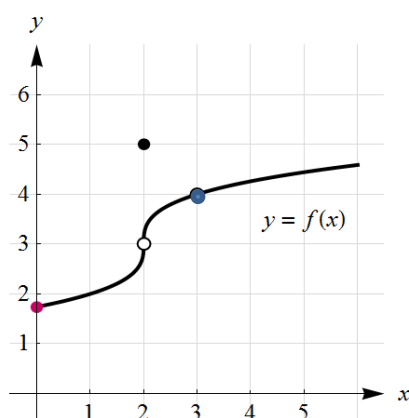
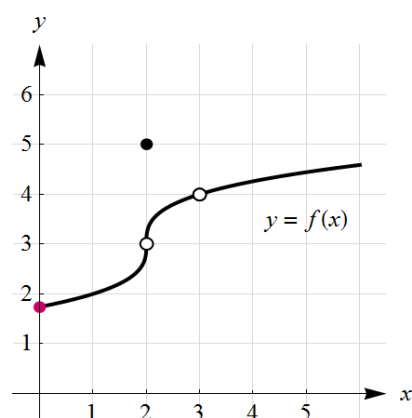
Warm-up:

Let $f(x) = \frac{x^2 - 6x + 8}{x - 2}$ and let $g(x) = x - 4$. Are f and g equivalent functions? Why or why not? Hint: Try factoring the numerator of $f(x)$.

Draw the graphs of $y = f(x)$ and $y = g(x)$.



Definitions



Limits:

There is a technical definition, but for us, the intuitive definition will do just fine. Here it is:

The limit L is the value the function “gets close to” if we make the x values “get close to” (but not equal to) a . We write $\lim_{x \rightarrow a} f(x) = L$.

Continuity:

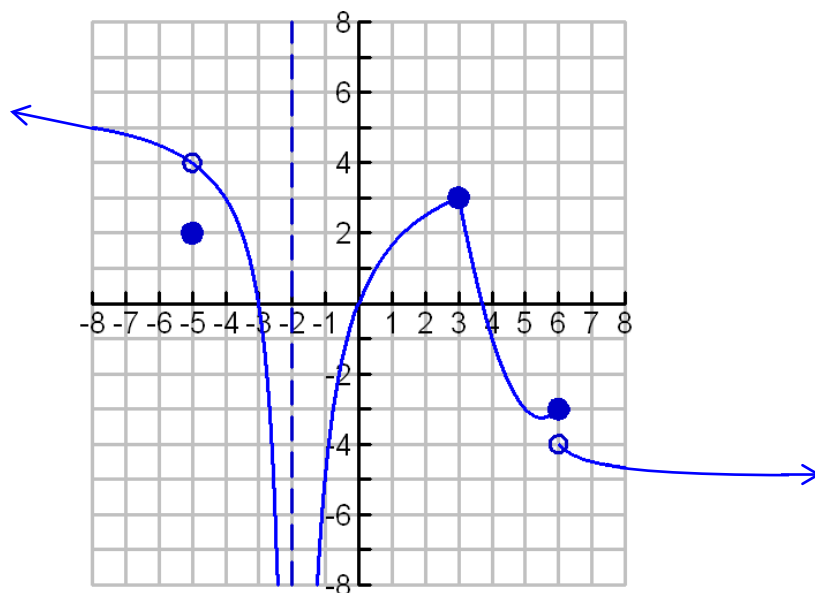
Definition: A function $f(x)$ is continuous at $x = a$ if and only if $\lim_{x \rightarrow a} f(x) = f(a)$.

This definition actually states three things:

- 1.
- 2.
- 3.

Part 1: Limits and Continuity given by a Graph

Let $y = f(x)$ be given by the graph below. Use the graph to find the following.



f. $\lim_{x \rightarrow -5} (4f(x)) =$

1. a. $f(1) =$

b. $\lim_{x \rightarrow 1} f(x) =$

c. Is f continuous at $x = 1$?

d. $\lim_{x \rightarrow 1} (f(x) + 3x) =$

2. a. $\lim_{x \rightarrow -2^-} f(x) =$

b. $\lim_{x \rightarrow -2^+} f(x) =$

c. $\lim_{x \rightarrow -2} f(x) =$

d. Is f continuous at $x = -2$?

3. a. $\lim_{x \rightarrow -5^-} f(x) =$

b. $\lim_{x \rightarrow -5^+} f(x) =$

c. $\lim_{x \rightarrow -5} f(x) =$

d. $f(-5) =$

e. Is f continuous at $x = -5$?

4. a. $\lim_{x \rightarrow 3^-} f(x) =$

b. $\lim_{x \rightarrow 3^+} f(x) =$

c. $\lim_{x \rightarrow 3} f(x) =$

d. $f(3) =$

e. Is f continuous at $x = 3$?

f. $\lim_{x \rightarrow 3} \sin\left(\frac{\pi}{3}f(x)\right) =$

5. a. $\lim_{x \rightarrow 6^-} f(x) =$

b. $\lim_{x \rightarrow 6^+} f(x) =$

c. $\lim_{x \rightarrow 6} f(x) =$

d. $f(6) =$

e. Is f continuous at $x = 6$?

f. $\lim_{x \rightarrow \infty} f(x) =$

g. $\lim_{x \rightarrow -\infty} f(x) =$

Part 2: Limits and Continuity given by an Equation

Hints about finding Limits:

- Plugging in nearby numbers will get you no credit unless the directions specifically say to do so.
- For any function which is CONTINUOUS, you can find the limit just by plugging in the number as long as the answer is defined.
 - In particular we know the following functions and all their combinations are continuous wherever they are defined:

_____	_____
_____	_____
_____	_____

- Ways you can combine continuous functions to get another continuous function:

_____	_____
_____	_____
_____	_____

- What do you do if the function is undefined? Look at the form of the limit!

$$\frac{\text{zero}}{\text{nonzero number}}$$

$$\frac{\text{nonzero number}}{\text{zero}}$$

$$\frac{\text{zero}}{\text{zero}}$$

$$\frac{\text{number}}{\text{infinity}}$$

$$\frac{\text{infinity}}{\text{infinity}}$$

Limits Problems:

a) $f(x) = \frac{1}{x-1}$

a) $\lim_{x \rightarrow 1^-} f(x) =$

b) $\lim_{x \rightarrow 1^+} f(x) =$

c) $\lim_{x \rightarrow 1} f(x) =$

d) $\lim_{x \rightarrow \infty} f(x) =$

- e) List the asymptotes of f . Justify your answers by citing appropriate limits. Explain how you know there are no others.

b) $f(x) = \frac{x^2-1}{x+1}$

a) $\lim_{x \rightarrow -1^-} f(x) =$

b) $\lim_{x \rightarrow -1^+} f(x) =$

c) $\lim_{x \rightarrow -1} f(x) =$

d) $\lim_{x \rightarrow \infty} f(x) =$

c) $h(x) = \frac{x^2-3x-4}{2x^2-4x-6} = \frac{(x-4)(x+1)}{2(x+1)(x-3)}$

a) $\lim_{x \rightarrow -1} h(x) =$

b) $\lim_{x \rightarrow 3} h(x) =$

c) $\lim_{x \rightarrow 4} h(x) =$

d) $\lim_{x \rightarrow \infty} h(x) =$

e) List the asymptotes of h . Justify your answers by citing appropriate limits. Explain how you know there are no others.

d) $h(x) = \begin{cases} -x^2 + 9, & x \leq -2 \\ -2x + 1, & -2 < x < 2 \\ x + 1, & x \geq 2 \end{cases}$

a) $\lim_{x \rightarrow -2} h(x) =$

b) $\lim_{x \rightarrow 2} h(x) =$

c) $\lim_{x \rightarrow 1} h(x) =$

e) $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1}-1}$

Comparison of Limits vs. Continuity

Limits

Conceptually

Where is the function headed (y-value) as you get near a certain x-value?

Graphically

No jumps or infinite squiggles, ignore the point itself

Algebraically

Limits from both sides have to agree

Math Notation

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$$

$f(x)$ is defined on an interval on both sides of a

Continuity

Conceptually

Can you draw it without picking up your pencil?

Graphically

No holes, breaks or infinite squiggles

Algebraically

1. Limits from both sides have to agree
2. The y-value of the point has to agree with the limit

Math Notation

1. $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$
2. $f(a)$ is defined
3. $f(a) = \lim_{x \rightarrow a} f(x)$

Squeeze Theorem

Let $f(x) = \sin(x)$. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \frac{f(x+1)}{2x^2}$$

Intermediate Value Theorem

If f is a continuous function on the closed interval $[a, b]$, and d is any value between $f(a)$ and $f(b)$, then there is a number c in (a, b) such that $f(c) = d$.

We will use the IVT to show that the equation

$$2 \log(x) = \frac{1}{\pi}$$

has a solution on the interval $(1, 10)$.

- a) IVT requires a single function, f . What is your choice for $f(x)$? _____
- b) **On which interval** do we need to show that f is continuous? _____
- c) Explain why f is continuous on that interval.

- d) Evaluate $f(1)$ and $f(10)$.
 $f(1) =$

 $f(10) =$
- e) IVT requires a number, d . What is your choice for d ? $d =$
- f) Fill in the blanks appropriately (according to your answers above): $f(\quad) < d < f(\quad)$
- g) Based on your answers above, explain how to use IVT to determine that the equation has a solution in the interval $(1, 10)$.