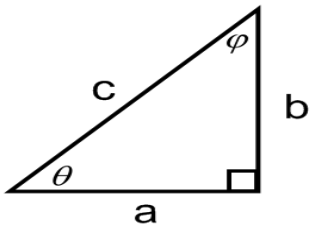
MSLC Workshop Series
Math 1149 Workshop: Inverse Trigonometric Functions

Angles versus Ratios of Numbers

In any use of trigonometry it is important to remember that there is a difference between the ratio of sides of the triangle and the reference angle, but when using inverse trig that difference is absolutely critical.

- Regular trig functions use reference angles to give a ratio of two sides of the triangle.
  - (Ex: $\cos\theta = \frac{a}{c}$)
- Inverse trig functions use a ratio of two sides of the triangle to give the reference angle.
  - (Ex: $\tan^{-1}\left(\frac{a}{b}\right) = \varphi$)



This means you shouldn't have a ratio of two numbers inside of a trig function, or an angle inside of an inverse trig function.

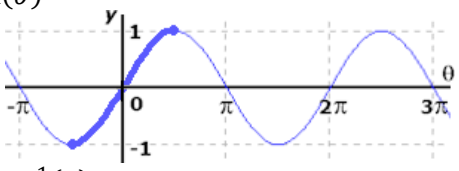
Domains and Ranges

All trigonometric functions are periodic (i.e. they repeat after a certain interval).

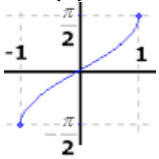
- The end result of this is trig functions aren't one-to-one.
- This causes problems for the inverse functions. The domain has to be restricted in order for the inverse to be a function.

$y = \arcsin x \quad (= \sin^{-1} x)$

$y = \sin(\theta)$



$y = \sin^{-1}(x)$

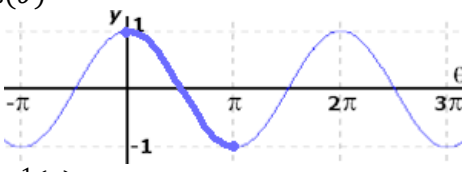


Domain: $-1 \leq x \leq 1$

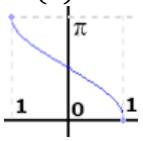
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \arccos x \quad (= \cos^{-1} x)$

$y = \cos(\theta)$



$y = \cos^{-1}(x)$

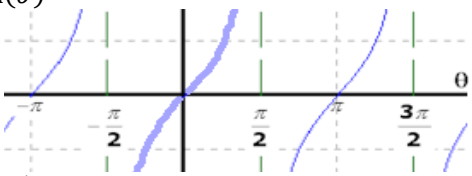


Domain: $-1 \leq x \leq 1$

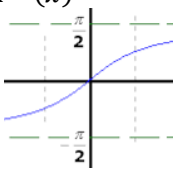
Range: $0 \leq y \leq \pi$

$y = \arctan x \; (= \cos^{-1} x)$

$y = \tan(\theta)$



$y = \tan^{-1}(x)$



Domain: $-\infty < x < \infty$

Range: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Boiled down, this means **inverse sine** and **inverse tangent** will give angles in **Quadrants I or IV** (specifically, θ will be between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$), and **inverse cosine** will give angles in **Quadrants I or II** (specifically, θ will be between 0 and π).

Simplify the following (*remember that* $\text{trig}^{-1}(r) = \theta$ *means* $\text{trig}(\theta) = r$)

1. $\arctan(\sqrt{3})$ 2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ 3. $\arccos\left(-\frac{1}{2}\right)$ 4. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Some more complex problems involve ***both trig and inverse trig functions.***

Inverse Trig (Trig)

1. $\sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right)$ 2. $\arcsin\left(\cos\left(-\frac{\pi}{2}\right)\right)$ 3. $\cos^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

Trig (Inverse Trig)

1. $\cos\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right)$ 2. $\tan\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$ 3. $\sin(\tan^{-1}x)$