# Compound Interest and Annuity Steps

Start at Step 1, then answer the questions and follow the corresponding steps. Descriptions of variables and explanations are at the end.

## Step 1: Is this an annuity?

Is the same size payment being made every period (e.g. every week, every month, every year)?

### No: Regular Compound interest

$$S = P(1 + r)^n$$

Where $r$ Is the periodic rate, $n$ is the total number of periods (see Variables and Explanation)

### Yes

See Step 2.

## Step 2: Present Value or Future Value?

### Present Value

Any loan, mortgage, cash-now price, or lottery (think: one amount in, many payments out). See Step 3.

### Future Value

Savings account, sinking fund, life insurance, or pension (think: many payments in, one amount out). See Step 4.

## Step 3: Is the payment at the end or beginning of each period?

### End: Present Value Ordinary Annuity

$$A = R\left[\frac{1 - (1 + r)^{-n}}{r}\right]$$

(angle notation $A = R \cdot \mathrm{a}_{\mathrm{n}|\mathrm{r}}$ where $r$ is the periodic rate, $n$ is the total number of periods.)

### Beginning: Present Value Annuity Due

$$A_{\text{due}} = R + R\left[\frac{1 - (1 + r)^{-(n-1)}}{r}\right]$$

(angle notation $A = R + R \cdot a_{n-1|r}$ where $r$ is the periodic rate, $n$ is the total number of periods.)

# Step 4: Is the payment at the end or beginning of each period?

## End: Future Value Ordinary Annuity

$$S = R\left[\frac{(1+r)^n - 1}{r}\right]$$

(angle notation: $S = R \cdot s_{n|r}$ where $r$ is the periodic rate, $n$ is the total number of periods.)

## Beginning: Future Value Annuity Due

$$S_{due} = R\left[\frac{(1+r)^{n+1} - 1}{r}\right] - R$$

(angle notation: $S = R \cdot s_{n+1|r} - R$ where $r$ is the periodic rate, $n$ is the total number of periods.)

# Variables and Explanation

$A$ = present value amount of the account

$S$ = future value amount of the account

$P$ = principal (beginning) amount

$R$ = periodic payment (must be equal)

$n$ = total # of compounding periods

$r$ = periodic interest rate

NOTE: $r$ and $n$ must have matching types (e.g. If $n$ is the total number of months, then $r$ must be the periodic monthly rate.) $r$ is usually given as the nominal rate, sometimes called APR (if the nominal rate is the annual percentage rate); the length of a nominal cycle is usually one year (certain rare businesses have a 2 year nominal cycle). The nominal rate can be adjusted by dividing $r$ by the number of periods in one nominal cycle, thus making $r$ and $n$ match in type.