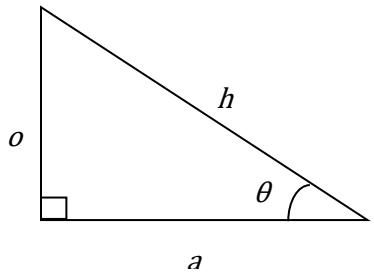
MSLC Trigonometry Handout

Trigonometric Functions

$$\sin\theta = \frac{o}{h} \qquad \csc\theta = \frac{h}{o}$$

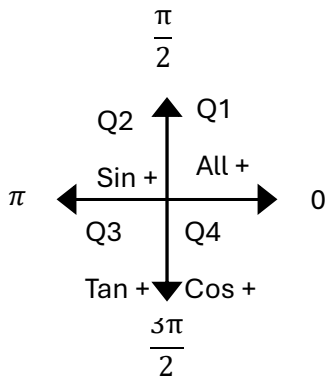
$$\cos\theta = \frac{a}{h} \qquad \sec\theta = \frac{h}{a}$$

$$\tan\theta = \frac{o}{a} \qquad \cot\theta = \frac{a}{o}$$



Trig Values for Special Angles

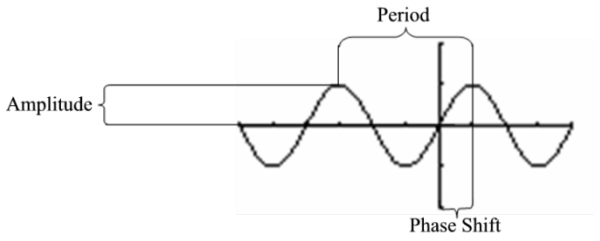
θ	**0** (0°)	**π/6** (30°)	**π/4** (45°)	**π/3** (60°)	**π/2** (90°)
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
$\cos\theta$	$1 = \frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined



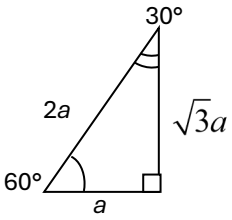
Graphing Trigonometric Functions

$$y = A\sin(Bx + C) \quad y = A\cos(Bx + C)$$

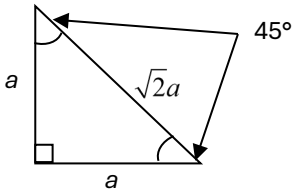
$$\text{Amplitude} = |A| \quad \text{Period} = \frac{2\pi}{B} \quad \text{PhaseShift} = \frac{-C}{B}$$



Special Triangles



30 – 60 – 90 Triangle: _____



45 – 45 – 90 Triangle: _____

Law of Sines

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Ratio Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Co-function Identities

$$\sin\theta = \cos(\pi/2 - \theta) \qquad \tan\theta = \cot(\pi/2 - \theta) \qquad \sec\theta = \csc(\pi/2 - \theta)$$

$$\cos\theta = \sin(\pi/2 - \theta) \qquad \cot\theta = \tan(\pi/2 - \theta) \qquad \csc\theta = \sec(\pi/2 - \theta)$$

Even/Odd Identities

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos\theta$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Addition and Subtraction Identities

$$\sin(x + y) = (\sin x)(\cos y) + (\sin y)(\cos x)$$

$$\sin(x - y) = (\sin x)(\cos y) - (\sin y)(\cos x)$$

$$\cos(x + y) = (\cos x)(\cos y) - (\sin x)(\sin y)$$

$$\cos(x - y) = (\cos x)(\cos y) + (\sin x)(\sin y)$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - (\tan x)(\tan y)}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + (\tan x)(\tan y)}$$

Double-Angle Identities

$$\sin(2x) = 2(\sin x)(\cos x)$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$$

Power Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Product-to-Sum Identities

$$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \sin y = \frac{1}{2}[\sin(x + y) - \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$$

Sum-to-Product Identities

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

Half-Angle Identities

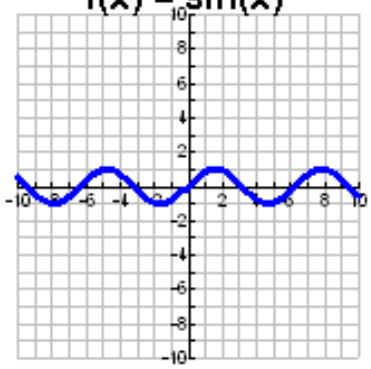
$$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

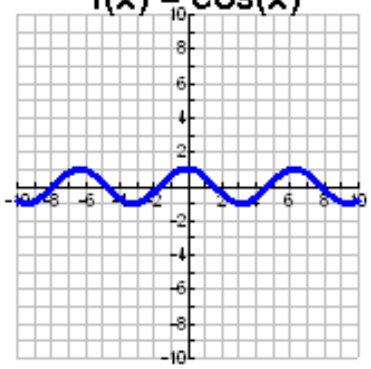
$$\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Trig Graphs

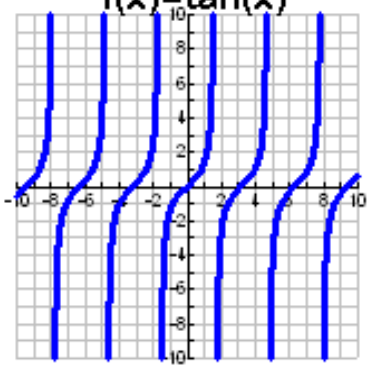
f(x) = sin(x)



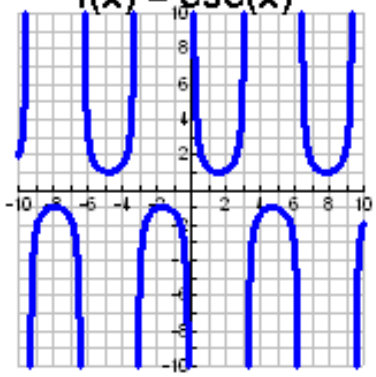
f(x) = cos(x)



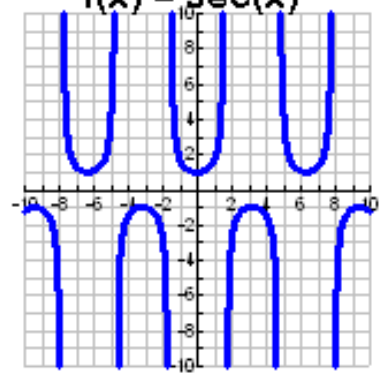
f(x)=tan(x)



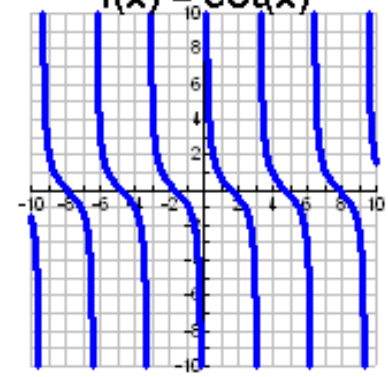
f(x) = csc(x)



f(x) = sec(x)



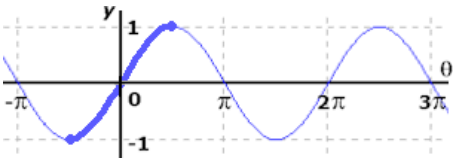
f(x) = cot(x)



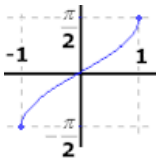
Inverse Trig Functions

$$y = \arcsin x \ (= \sin^{-1} x)$$

$y = \sin(\theta)$



$y = \sin^{-1}(x)$

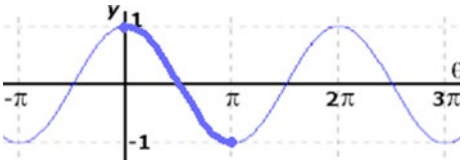


Domain: $-1 \leq x \leq 1$

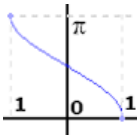
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$y = \arccos x \ (= \cos^{-1} x)$$

$y = \cos(\theta)$



$y = \cos^{-1}(x)$

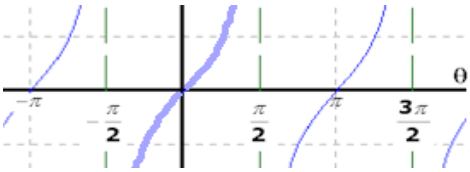


Domain: $-1 \leq x \leq 1$

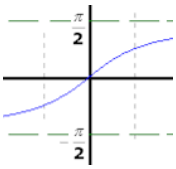
Range: $0 \leq y \leq \pi$

$$y = \arctan x \ (= \tan^{-1} x)$$

$y = \tan(\theta)$



$y = \tan^{-1}(x)$



Domain: $-\infty \leq x \leq \infty$

Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Radians and Degrees

Converting Radians to Degrees: $\theta \cdot \frac{180}{\pi}$

Converting Degrees to Radians: $\theta \cdot \frac{\pi}{180}$

The Unit Circle