Summary of Limits, Continuity and Differentiability

Limits

Conceptually

Where is the function headed (y-value) as you get near a certain x-value?

Graphically

No jumps or infinite squiggles, ignore the point itself

Algebraically

Limits from both sides have to agree

Math Notation

$$\lim_{x \to a^+} f\,(x) = \lim_{x \to a^-} f\,(x)$$

$f(x)$ is defined on an interval on both sides of a

Continuity

Conceptually

Can you draw it without picking up your pencil?

Graphically

No holes, breaks or infinite squiggles

Algebraically

1. Limits from both sides have to agree
2. The y-value of the point has to agree with the limit

Math Notation

1. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$
2. $f(\mathrm{a})$ is defined
3. $f(a) = \lim_{x \to a} f(x)$

Differentiability

Conceptually

Is it smooth?

Graphically

No corners, breaks or infinite squiggles

Algebraically

1. Limits from both sides have to agree
2. The y-value of the point has to agree with the limit
3. Limit of the difference quotient must also exist

Math Notation

1. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$
2. $f(\mathrm{a})$ is defined and $f(a) = \lim_{x \to a} f(x)$
3. $\lim_{h \to 0^+} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0^-} \frac{f(x+h)-f(x)}{h}$