MSLC Workshop Series
Math 1151 – Workshop
The Fundamental Theorem of Calculus and Substitution

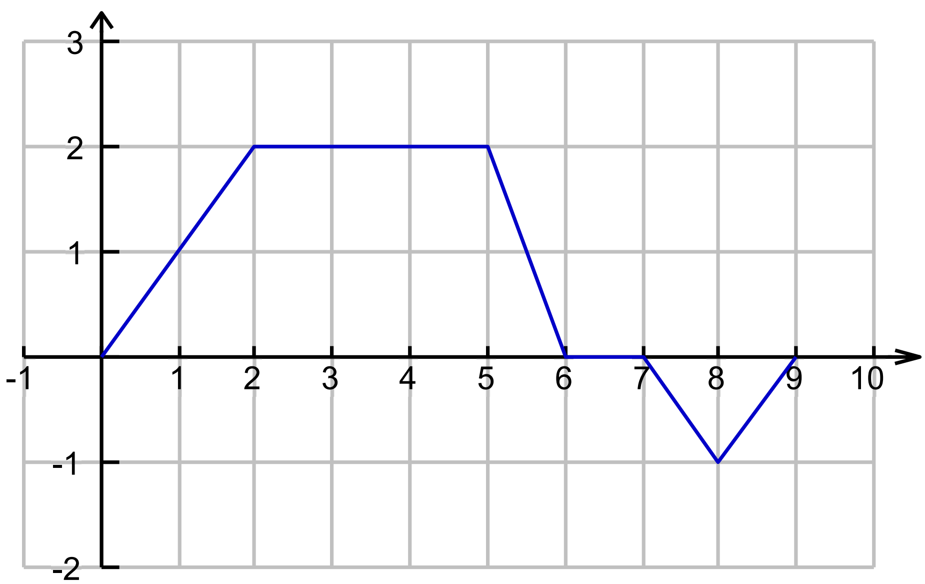
First Fundamental Theorem of Calculus

- $\int_a^b f(t)dt$ calculates the signed area under the curve $y = f(t)$ from a to b.
- Similarly, we can find the area under the curve between a and x (where x is a variable) with the integral $\int_a^x f(t)dt$.
- This integral can be thought of as a function of x! Let's call it:

$$A(x) = \int_a^x f(t)dt$$

(This function is called the *accumulation function* of the original function, $f(x)$.)

Assume $a = 0$ and $f(x)$ is the function below.



1. Find $A(2)$.
2. Find $A(9)$.
3. Is A increasing or decreasing on the interval $[5,6]$?

The rate of accumulation at $t = x$ is equal to the value of the function being accumulated at $t = x$. This relationship is known as the **First Fundamental Theorem of Calculus**. That is,

$A'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$, where $f(t)$ is a continuous function on $[a, x]$.

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Examples:

Find $F'(x)$.

1. $F(x) = \int_2^x (t^2 - \cos t + 3)dt$

2. $F(x) = \int_0^{x^2} t\cos(2t)\,dt$

3. $F(x) = \int_x^4 \frac{3}{t}\,dt$

4. $F(x) = \int_x^{x^3} \sin^2(t)\,dt$

Second Fundamental Theorem of Calculus

What if the function we are integrating doesn't lend itself to nice geometry calculation?

- We could use Riemann Sums

- OR -

- We could use the Second Fundamental Theorem, which states

$\int_a^b f(x)dx = F(b) - F(a)$, where $F(x)$ is <u>any</u> antiderivative of $f(x)$.

(This is much more efficient than using Riemann sums, assuming we can find an antiderivative of the function.)

Examples:

1. $\int_1^3 (2x^2 + x)\, dx$

2. $\int_0^{\frac{\pi}{4}} \sec^2(x)\, dx$

Substitution – Undoing the Chain Rule

Suppose we want to find $\int \sin(x^4)\,4x^3 dx.$ We know how to find $\int \sin(\blacksquare)\, d\blacksquare$, so we are going to do a **CHANGE OF VARIABLE**.

$$u = x^4$$

But to do this, we also need to change the dx into a du.

$$\frac{du}{dx} =$$

Which gives us a new integral that we know how to solve:

CONVERTING ALL x's to u's

Most importantly, this change of variable will only work if we change EVERY x so that the ENTIRE integral is in terms of u.

When Substitution Fails:

$\int \sin(x^4)\, dx$

Back Substitution:

$\int \frac{x}{x+1} dx$

Limits of Integration:

$\int_1^2 (x+3)^{60} dx$

Practice Problems:

1. $\int_{\pi/4}^{\pi} \sin x \cos x \, dx$

2. $\int_0^4 \sqrt{2t+1} dt$

3. $\int_0^4 x^2 \cos(x^3 - 1) \, dx$

4. $\int_{-4}^4 \sqrt{16 - x^2} dx$ (Hint: remember integral means area)

5. $\int_{-4}^{4} \frac{2}{1+4x^2}\, dx$

6. $\int_{-4}^{2} \sin{(x+1)} dx$

7. $\int_{0}^{1} 5x^2 e^{x^3}\, dx$

8. $\int \frac{2x+1}{x^2+4} dx$