MSLC Workshop Series

Math 1151 – Workshop

Sigma Notation and Riemann Sums

**Sigma Notation:**

Notation and Interpretation of 

* (capital Greek sigma, corresponds to the letter S) indicates that we are to **sum** numbers of the form indicated by the general term
*  is the **general term**, which determines what is being summed, and can be defined however we want but is usually a formula containing the index: 
* *k* is called the **index**; we may use any letter for the index, typically we use i, j, k, l, m, and n as indices
* The index runs through the positive integers, starting with the number below the (in this case 1) and ending with the integer above the (in this case *n*)
* The sum on the right-hand side is the **expanded form**. (The  contains all the terms I was too lazy to write.)
* The letter below the sigma is the variable with respect to the sum. All other letters are **constants** with respect to the sum.

Examples:

1.  2.  3. 

Examples:

Special Sum Formulas

Properties of Sigma Notation is an **operator** that represents summation, and its properties are similar to the properties of addition (note what properties are **not** mentioned here).

* Multiplication by a common constant (also called a ***scalar*** multiple) 
* Addition or Subtraction (this is also called the ***linearity*** property) 

Example:

**Riemann Sums:** 



**Definition of a Riemann Sum:**

Consider a function  defined on a closed interval , partitioned into  subintervals of equal width by means of grid points . On each subinterval , pick a sample point . Then the Riemann sum for  corresponding to this partition is given by:



* **WIDTH:** 

Since we partition the interval into evenly spaced partitions, we can calculate the width:

, where *n* is the number of partitions.

* **HEIGHT:** 

Also, we usually don’t pick  arbitrarily. We use a rule to pick . The most common rules to use are the Right Endpoint Rule, the Left Endpoint Rule, and the Midpoint Rule.

The most common rules to use are:

Right Endpoint Rule 

Left Endpoint Rule 

Midpoint Rule 

* If we partition the interval into more and more rectangles with smaller and smaller widths, we get closer to the (signed) area trapped between the curve  and the -axis.

This is where **Sigma Notation** comes in because it becomes time consuming to add up all the terms when there are many, many rectangles.

**Calculating A Riemann Sum**

Using the **Right Endpoint Rule**, the Riemann sum becomes:



Using the **Left Endpoint Rule**, the Riemann sum becomes:



Using the **Midpoint Rule**, the Riemann sum becomes:



**Example:**

Estimate the area under on the interval [-2, 3] using right Riemann Sums and 5 rectangles. No need to use sigma notation here.



Estimate the area under on the interval [-2, 3] using left Riemann Sums and 5 rectangles.



Estimate the area under on the interval using midpoint Riemann Sums and 5 rectangles.



**Example:** Estimate the area under on the interval [0, 2] using **right Riemann sums** and 10 rectangles. Try using sigma notation!

First calculate the width: 

Then the x-value for the right endpoint of the *k*th rectangle is

Thus the height of the *k*th rectangle is

So the Riemann sum is

Now evaluate this sum using your knowledge of sigma notation!

**Example:** Estimate the area under on the interval [0, 2] using **right Riemann sums** and 50 rectangles. Try to use your work from the previous problem. **What changes?**

First calculate the width:

Then the x-value for the right endpoint of the *k*th rectangle is

Thus the height of the *k*th rectangle is

So the Riemann sum is

Now evaluate this sum using your knowledge of sigma notation!

**Being More Accurate:**

What if we want to get a better approximation than any of the above give us? More rectangles will give us less extra or unused area between the curve and the -axis. Let’s again do a **right sum** for on the interval :

using equal subintervals.

First calculate the width: 

Then the x-value for the right endpoint of the *k*th rectangle is

Thus the height of the *k*th rectangle is

So the Riemann sum is

Now evaluate this sum using your knowledge of sigma notation!

If you have time, take the limit as goes to infinity! This is the definite integral .

Note that the definite integral can often be calculated as an area using geometry or with the fundamental theorem of calculus! Don’t do more work than you have to!