MSLC Workshop Series
Math 1151 – Workshop
Optimization

Which are you trying to find in an optimization problem?

What is a critical point and how do you find one?

What is the difference between a local min/max and an absolute (global) min/max?

How do you find an absolute maximum/minimum on a**…**

Closed interval	Open interval if there is only one critical point

Closed Interval [a, b]

The global max and min must be attained somewhere on the interval by the Extreme Value Theorem. Therefore, we need to find the function value at critical points and the endpoints of the interval a and b, then compare them to see which is largest/smallest.

Open interval if there is only one critical point

A local max/min is the global max/min. We can use the first or second derivative test to find out whether it is a max or a min.

Derivative Tests

For the **first derivative test**, make a sign chart for f' around the critical point.

- If f' goes from positive to negative, f goes from increasing to decreasing, and you've found a local max.
- If f' goes from negative to positive, f goes from decreasing to increasing, and you've found a local min.

For the **second derivative test**, find f'' at the critical point.

- If f'' is positive, f is concave up, and you've found a local min.
- If f'' is negative, f is concave down, and you've found a local max.

<u>Example 1:</u>

A farmer with 750 feet of fencing wants to enclose a rectangular area and divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

How do we translate the problem into math?

How can we relate the important quantities?

Optimization equation—what do you want to maximize/minimize?

Constraint equation(s)—what stops you from building as big an area as you want?

How can we do calculus? We need an equation with one variable!

What is your interval of interest? How does it affect your problem-solving method?

Using the previous information, apply the appropriate method for finding the absolute max/min.

What quantity is the question asking for? Units are also important.

Example 2:

A rectangular storage container with an open top is to have a volume of $10m^3$. The length of its base is twice the width. Material for the sides costs $6 per square meter. Material for the base costs $10 per square meter. Find the cost of the cheapest such container.

How do we translate the problem into math?

How can we relate the important quantities?

Optimization equation—what do you want to maximize/minimize?

Constraint equation(s)—what stops you from building as big an area as you want?

How can we do calculus? We need an equation with one variable!

What is your interval of interest? How does it affect your problem-solving method?

Using the previous information, apply the appropriate method for finding the absolute max/min.

What quantity is the question asking for? Units are also important.

Practice Problems:

1. Liz is standing on the bank of a 30-foot-wide river. An ice cream shop is located 150 feet down river on the opposite bank of the river. Liz plans to get to the ice cream shop by a combination of swimming across the river to a point x feet downriver on the opposite bank and jogging the rest of the way along the river. She can swim at a rate of 6 feet per second and can jog at a rate of 7 feet per second. To what point on the opposite bank should she swim to reach the ice cream shop in the least amount of time? Feel free to use a calculator.

2. Find the point (x, y) on the graph of $y = \sqrt{x}$ nearest the point (4, 0).

Answers:

1. $\frac{180}{\sqrt{13}}$ feet downriver 2. $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$

Thought Process For Optimization Problems

1. **How did we turn the problem into mathematics?**
   - Draw a picture, paying careful attention to the problem to ensure all relevant details are present and accurate.
   - Label your diagram, so we can **write down equations**.
   - The ***optimization equation*** is the one you want to maximize or minimize. Pay attention to what the story wants you to optimize.
   - The ***constraint equation(s)*** are what prevent your answer from being as big (or small) as it wants.

2. **Why did we reduce the optimization equation to a function of only one variable?**
   Do this so we can use our calculus techniques on the optimization equation!

3. **How did we find our interval and determine if it is open or closed? How did we decide which method we want to use?**
   - We considered all values of the variable that made sense, then chose the method based on whether our interval was open or closed.
4. **Use your method: First find critical points, then**
   - **Closed interval:**
     i. Plug Critical Points and End Points into the optimization equation.
     ii. The biggest value is the absolute max; the smallest is the absolute min.
   - **Open interval, 1 critical point:**
     i. The critical point is guaranteed to be an absolute max or min. To determine which, decide if it is a local max or min.
     ii. Use the first or second derivative test for maxima and minima.
5. **Did you answer the question?**
   Reread the original problem as look at what number they want as the answer. It may be the max/min value, but it could also be the variable value that produces the max/min.