MSLC Workshop Series
Math 1151 – Workshop
Indeterminate Forms and L'Hopital's Rule

INDETERMINATE FORMS

What does it mean for a limit's form to be indeterminate?

Which forms are indeterminate?
Circle the forms which are indeterminate.

$\infty \cdot \infty$	$0 \cdot \infty$	$\# \cdot \infty$
$\infty + \infty$	$\infty - \infty$	$\infty + \#$
$\frac{\infty}{\infty}$	$\frac{0}{\infty}$	$\frac{\#}{\infty}$
$\frac{\#}{0}$	$\frac{0}{0}$	$\frac{\infty}{0}$
1^0	1^∞	∞^0
0^0	0^∞	∞^∞

Determining the Form Exercises:

Write the form of each of the following limits.

1. $\lim\limits_{x \to 0^+} (\sin(x)\cot(x))$

2. $\lim\limits_{x \to \infty} \left(\frac{\arctan(x)}{x}\right)$

3. $\lim\limits_{x \to \infty} \left(\left[\frac{1}{x} + 1\right]^x\right)$

4. $\lim\limits_{x \to \infty} (e^x - x)$

5. $\lim\limits_{x \to \infty} \left(\frac{1}{x} + 1\right)^{\frac{1}{x}}$

6. $\lim\limits_{x \to \infty} ([\ln(1 + e^{-x})]^x)$

7. $\lim\limits_{x \to \infty} (e^x + x)$

8. $\lim\limits_{x \to \infty} \left(\frac{3x^2+5x-2}{6x^2+3x+1}\right)$

9. $\lim\limits_{x \to \frac{\pi}{2}^-} \left(\frac{\cos(x)}{\tan(x)}\right)$

10. $\lim\limits_{x \to \infty} \left(x \ln\left(\frac{1}{x}\right)\right)$

11. $\lim\limits_{x \to \infty} \left([\operatorname{arccot}(x) + \pi]^{\frac{1}{x}}\right)$

12. $\lim\limits_{x \to 1} \left(\frac{x^2-1}{5\ln(x)}\right)$

13. $\lim\limits_{x \to 3} \left(\frac{x^2+2}{x^2-x-6}\right)$

14. $\lim\limits_{x \to 1^-} (\cot(\pi x) + \sec(x))$

L'HOPITAL'S RULE

L'Hopital's rule is a way of dealing with indeterminate forms of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

L'Hopital's Rule: If you want to know $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$,

and IF: #1. both $\lim\limits_{x\to a} f(x)=0$ and $\lim\limits_{x\to a} g(x)=0$

- OR -

#2. both $\lim\limits_{x\to a} f(x)=\infty$ and $\lim\limits_{x\to a} g(x)=\infty$,

THEN: $\lim\limits_{x\to a}\frac{f(x)}{g(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$.

Notes:

1. **This is NOT the quotient rule for derivatives**
2. **L'Hopital's Rule give the WRONG answer if #1 or #2 is not satisfied.**

Examples:

$$\lim_{x\to 1}\left(\frac{x^2-1}{5\ln x}\right)=$$

$$\lim_{x\to\infty}\left(\frac{3x^2+5x-2}{6x^2+3x+1}\right)=$$

***<u>L'Hopital's Rule Exercises</u>*:**

Evaluate the following limits, using L'Hopital's Rule ***if appropriate***

1. $\lim_{x\to 0}\frac{e^x-3x-1}{5x}$

2. $\lim_{x\to 0}\frac{\sin^2 x}{\cos x-1}$

3. $\lim_{x\to 0}\frac{e^x}{x^2}$

4. $\lim_{x\to \infty}\frac{e^x}{x^2}$

5. $\lim_{x\to \frac{\pi}{2}}\frac{\tan(x)}{\csc(x)}=$

FORCING A FRACTION

L'Hopital can also help us with the other indeterminate forms if we can **force a fraction** that yields $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Examples:

1. $\lim_{x\to\infty}\left(x\tan\frac{1}{x}\right)=$

2. $\lim_{x\to\infty}([\frac{1}{x}+1]^x)=$

3. $\lim_{x\to\infty}(x-\ln x)=$

Forcing a Fraction Exercises:
Evaluate the following by applying L'Hopital's Rule ***if appropriate***.

1. $\lim_{x\to 0^+} x\ln 3x$

2. $\lim_{x\to\infty} e^{-x}\sqrt{x}$

3. $\lim_{x\to\infty}([\cos\left(\frac{2}{x}\right)]^{x^2}) =$

4. $\lim_{x\to 3}\frac{e^x}{x^2-9} =$

5. $\lim_{x\to\infty}(\csc(x)-\frac{1}{x}) =$

6. $\lim_{x\to 0^-}\left(\frac{1}{x^2}+\frac{\cos(3x)}{x^3}\right) =$