MSLC Workshop Series
Math 1151 - Workshop
Derivative "Short-Cut" Rules

There are certain functions that you should know how to take the derivative of.
Make sure you have these memorized! We won't go over them together, but try to fill out this page on your own!

A CONSTANT: $\boxed{\frac{d}{dx}(c)=0}$ [1]

Examples: $\frac{d}{dx}(42)=$ $\frac{d}{dx}\left(\sqrt{3}\right)=$ $\frac{d}{dx}(-\pi)=$

X TO A CONSTANT POWER: $\boxed{\frac{d}{dx}\left(x^n\right)=nx^{n-1}}$ [2]

Examples: $\frac{d}{dx}\left(x^5\right)=$ $\frac{d}{dx}\left(\frac{1}{x^3}\right)=$ $\frac{d}{dx}\left(\sqrt[3]{x}\right)=$

THE TRIG DERIVATIVES:

$\frac{d}{dx}(\sin x)=$ $\frac{d}{dx}(\cos x)=$

$\frac{d}{dx}(\sec x)=$ $\frac{d}{dx}(\csc x)=$

$\frac{d}{dx}(\tan x)=$ $\frac{d}{dx}(\cot x)=$

DERIVATIVES OF EXPONENTIAL FUNCTIONS:

$\frac{d}{dx}\left(e^x\right)=$ $\frac{d}{dx}\left(a^x\right)=$ $\frac{d}{dx}\left(x^x\right)=$

$\frac{d}{dx}(\ln x)=$ $\frac{d}{dx}(\log_b x)=$

DERIVATIVES OF INVERSE TRIG FUNCTIONS:

$$\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}} \qquad (-1<x<1)$$

$$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x)=-\frac{1}{1+x^2} \qquad (-\infty<x<\infty)$$

$$\frac{d}{dx}(\sec^{-1}x)=\frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}(\csc^{-1}x)=-\frac{1}{|x|\sqrt{x^2-1}} \qquad (x<-1, x>1)$$

IMPORTANT: These rules ONLY work when for these exact functions with a single variable (i.e. x) plugged in to them!

[1] c is a constant (no x's allowed!)
[2] n is a constant (no x's allowed!)

What do we do if we can see that we have more than one of these basic functions to differentiate in the same problem? Well, that depends entirely on how the functions are combined.

Here are the ways the basic functions can be combined and what to do about it:

The functions are combined by:

The constant multiple rule:

$$\boxed{\left(c \cdot f(x)\right)' = c \cdot f'(x)}$$ [3]

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Example: If $y = \pi x^2$, then $y' =$

The sum/difference rule:

$$\boxed{(f \pm g)' = f' \pm g'}$$

Example: If $y = 3x^4 - 6x^3 + 7x^{3/2}$, then $y' =$ __________________________

The product rule:

$$\boxed{(f \cdot g)' = f' \cdot g + f \cdot g'}$$

Example: If $y = \left(3x^2 + 2x - 7\right)\left(4x^8 - 10x + 3\right)$, then $y' =$ __________________________

The quotient rule:

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}}$$

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Example: Find $\frac{dy}{dx}$ if $y = \frac{x^2 + 3x - 5}{x^4 + 9x}$.

[3] c is a constant (no x's allowed!)

The functions are combined by:

The Chain Rule:

$$\boxed{\left[f\left(g(x)\right)\right]' = f'\left[g\left(x\right)\right]\cdot g'\left(x\right)}$$

Alternate Notation #1: $\frac{d}{dx}(f\circ g)(x) = f'\left(g\left(x\right)\right)\cdot g'\left(x\right)$

Alternate Notation #2: $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$, where $y = f\left(u\right)$ and $u = g\left(x\right)$

Examples of the Chain Rule:

$$\frac{d}{dx}\left(\left(x^5 - 3x\right)^{57}\right) =$$

$$\frac{d}{dx}\left(e^{x\cot x}\right) =$$

$$\left(\cos^3\left(x\right)\right)' =$$

If $f(x) = \sin\left(\sqrt{3x^4 - \pi x}\right)$,

then $f'\left(x\right) =$

The best thing you can do now is practice finding lots of derivatives using all the rules until they become familiar. Find $\frac{dy}{dx}$ for the following. To start, try to find the outermost operation (addition, division, composition, etc.). Do not simplify the results. **Treat a and m as constants, and f and g as differentiable functions.**

1. $y = x + \ln(x^2)$

2. $y = \frac{2x}{e(x-1)}$

3. $y = \frac{\pi x - 3}{(3x^2 - 4x)^3}$

4. $y = \sec(x^3 3^x)$

5. $y = \sec^3(x)3^x + x^3 3^x$

6. $y = \sqrt[3]{(3x^3 - 5x^2)}\sin(mx) + \sqrt{(\pi + 4)}$

7. $y = \cos^3\left(\frac{f(x)x^2}{1-x}\right)$

8. $y = \tan^{-1}\left(e^{x^2}\right)$

9. $y = \sqrt[4]{xa^x + 3e^3}$

10. $y = \csc\left(e^{\sec(xg(x))}\right)$