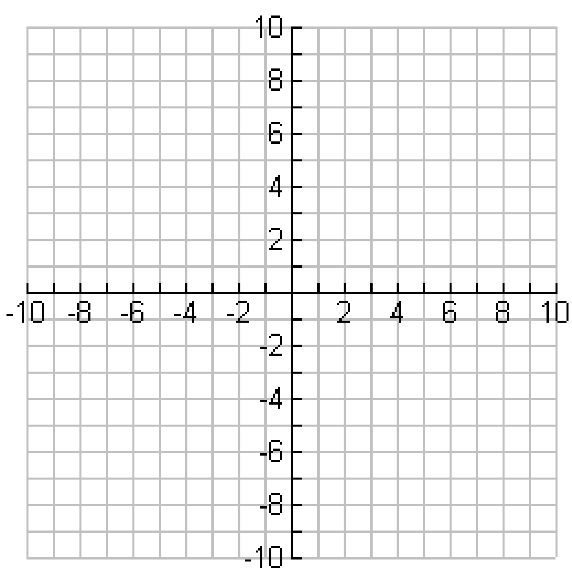
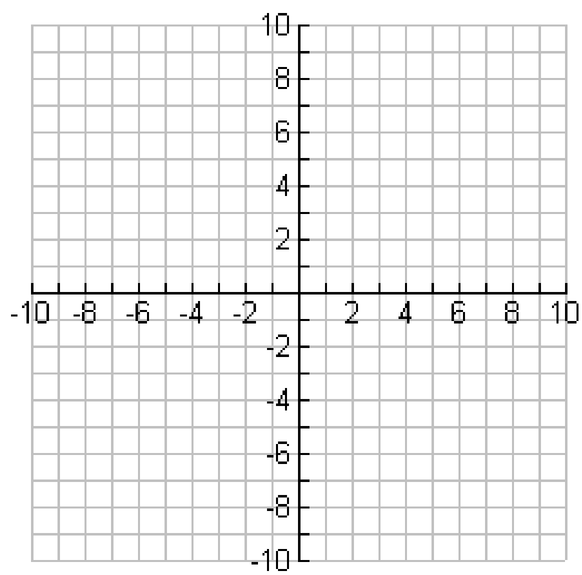
**MSLC Workshop Series**
**Math 1151 – Workshop 1**
**Limits and Continuity**

Warm-up 1: Let $f(x) = \frac{x^2-6x+8}{x-2}$ and let $g(x) = x - 4$. Are $f$ and $g$ equivalent functions? Why or why not? Hint: Try factoring the numerator of $f(x)$.

Draw the graphs of $y = f(x)$ and $y = g(x)$.



## **Limits:**

There is a technical definition, but for us, the intuitive definition will do just fine. Here it is:

The limit $L$ is the value the function "gets close to" if we make the *x* values "get close to" (but not equal to) $a$. We write $\lim_{x \to a} f(x) = L$.

## **Continuity:**

**Definition:** A function $f(x)$ is continuous at $x = a$ if and only if $\lim_{x \to a} f(x) = f(a)$.

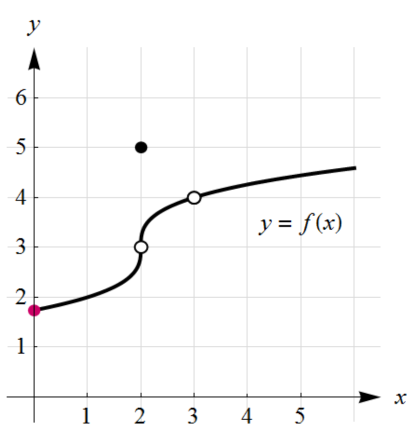
This definition actually states three things:

1.

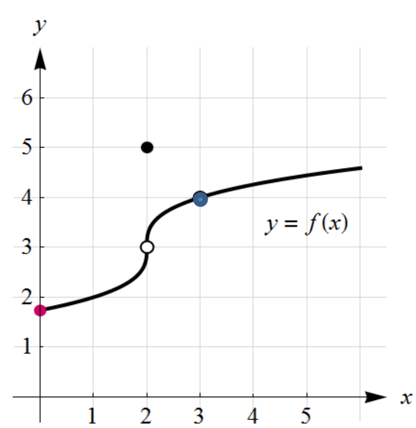
2.

3.

## **Limits**




## **Continuity:**

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*Part 1: Limits and Continuity given by a Graph*

Let $y = f(x)$ be given by the graph below. Use the graph to find the following.

1. a. $f(1) =$

   b. $\lim_{x\to 1} f(x) =$

   c. Is $f$ continuous at $x = 1$?

   d. $\lim_{x\to 1}(f(x) + 3x) =$

2. a. $\lim_{x\to -2^-} f(x) =$

   b. $\lim_{x\to -2^+} f(x) =$

   c. $\lim_{x\to -2} f(x) =$

   d. Is $f$ continuous at $x = -2$?

3. a. $\lim_{x\to -5^-} f(x) =$

   b. $\lim_{x\to -5^+} f(x) =$

   c. $\lim_{x\to -5} f(x) =$

   d. $f(-5) =$

   e. Is $f$ continuous at $x = -5$?

   f. $\lim_{x\to -5}(4f(x)) =$

4. a. $\lim_{x\to 3^-} f(x) =$

   b. $\lim_{x\to 3^+} f(x) =$

   c. $\lim_{x\to 3} f(x) =$

   d. $f(3) =$

   e. Is $f$ continuous at $x = 3$?

   f. $\lim_{x\to 3} \sin\left(\frac{\pi}{3} f(x)\right) =$

5. a. $\lim_{x\to 6^-} f(x) =$

   b. $\lim_{x\to 6^+} f(x) =$

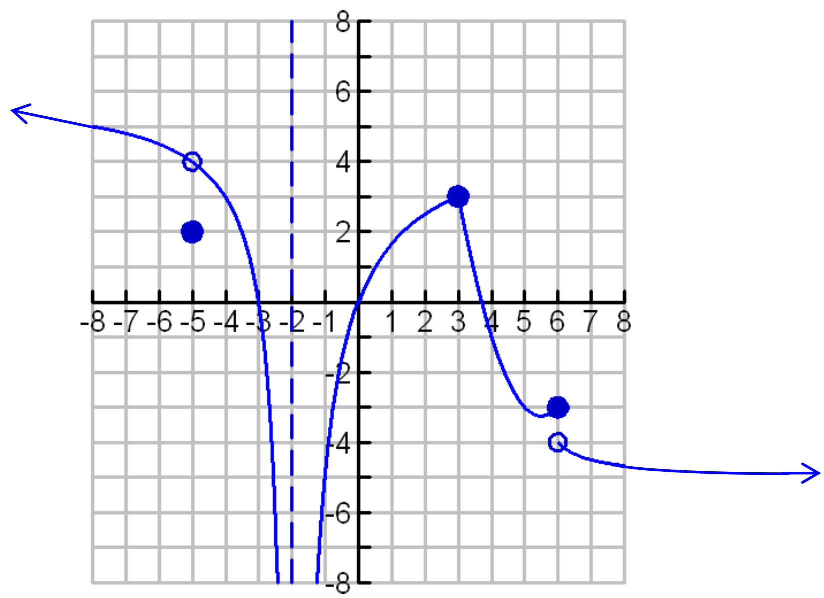
   c. $\lim_{x\to 6} f(x) =$

   d. $f(6) =$

   e. Is $f$ continuous at $x = 6$?

   f. $\lim_{x\to \infty} f(x) =$

   g. $\lim_{x\to -\infty} f(x) =$



*Part 2: Limits and Continuity given by an Equation*

**Hints about finding Limits:**

- Plugging in nearby numbers will get you no credit unless the directions specifically say to do so.
- For any function which is CONTINUOUS, you can find the limit just by plugging in the number as long as the answer is defined.
  - In particular we know the following functions and all their combinations are continuous wherever they are defined:

    ______________________________ ______________________________

    ______________________________ ______________________________

    ______________________________ ______________________________

    ______________________________

  - Ways you can combine continuous functions to get another continuous function:

    ______________________________ ______________________________

    ______________________________ ______________________________

    ______________________________ ______________________________

- What do you do if the function is undefined? Look at the form of the limit!

$\dfrac{\text{zero}}{\text{nonzero number}}$ $\dfrac{\text{nonzero number}}{\text{zero}}$ $\dfrac{\text{zero}}{\text{zero}}$

$\dfrac{\text{number}}{\text{infinity}}$ $\dfrac{\text{infinity}}{\text{infinity}}$

Limits Problems:

a) $f(x)=\frac{1}{x-1}$

a) $\lim_{x\to 1^-} f(x)=$

b) $\lim_{x\to 1^+} f(x)=$

c) $\lim_{x\to 1} f(x)=$

d) $\lim_{x\to \infty} f(x)=$

e) List the asymptotes of $f$. Justify your answers by citing appropriate limits. Explain how you know there are no others.

b) $f(x)=\frac{x^2-1}{x+1}$

a) $\lim_{x\to -1^-} f(x)=$

b) $\lim_{x\to -1^+} f(x)=$

c) $\lim_{x\to -1} f(x)=$

d) $\lim_{x\to \infty} f(x)=$

c) $h(x)=\frac{x^2-3x-4}{2x^2-4x-6}=\frac{(x-4)(x+1)}{2(x+1)(x-3)}$

a) $\lim_{x\to -1} f(x)=$

b) $\lim_{x\to 3} f(x)=$

c) $\lim_{x \to 4} f(x) =$

d) $\lim_{x \to \infty} f(x) =$

e) List the asymptotes of $f$. Justify your answers by citing appropriate limits. Explain how you know there are no others.

d) $h(x) = \begin{cases} -x^2 + 9, & x \le -2 \\ -2x + 1, & -2 < x < 2 \\ x + 1 & x \ge 2 \end{cases}$

a) $\lim_{x \to -2} f(x) =$

b) $\lim_{x \to 2} f(x) =$

c) $\lim_{x \to 1} f(x) =$

e) $\lim_{x \to 0} \dfrac{x}{\sqrt{x+1} - 1}$

# Comparison Chart of Limits vs. Continuity

| | Limits | Continuity |
|---|---|---|
| **Conceptually** | Where is the function headed (y-value) as you get near a certain x-value? | Can you draw it without picking up your pencil? |
| **Graphically** | No jumps or infinite squiggles, ignore the point itself | No holes, breaks, or infinite squiggles |
| **Algebraically** | 1) Limits from both sides have to agree | 1) Limits from both sides have to agree<br>2) The y-value of the point has to agree with the limit |
| **Math Notation**<br>* And fine print | 1) $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$<br>*f(x) is defined on an interval on both sides of a | 1) $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$<br>2) $f(a)$ is defined and $f(a) = \lim_{x \to a} f(x)$ |

# Squeeze Theorem

Let $f(x) = \sin(x)$. Evaluate the limit:

$$\lim_{x \to \infty} \frac{f(x+1)}{2x^2}$$

# Intermediate Value Theorem

If $f$ is a continuous function on the closed interval $[a, b]$, and $d$ is any value between $f(a)$ and $f(b)$, then there is a number $c$ in $(a, b)$ such that $f(c) = d$.

We will use the IVT to show that the equation

$$2\log(x) = \frac{1}{\pi}$$

has a solution on the interval (1, 10).

a) IVT requires a single function, $f$. What is your choice for $f(x)$? ____________________________

b) **On which interval** do we need to show that $f$ is continuous? ______________________________

c) Explain why $f$ is continuous on that interval.

d) Evaluate $f(1)$ and $f(10)$.

$f(1) =$ ___________________

$f(10) =$ ________________

e) IVT requires a number, $d$. What is your choice for $d$? $d =$ ___________________

f) Fill in the blanks appropriately (according to your answers above): $f(\quad) < d < f(\quad)$

g) Based on your answers above, explain how to use IVT to determine that the equation has a solution in the interval (1, 10).