MSLC Workshop Series
Math 1149 & 1150: Trigonometric Equations

The goal of this workshop is to familiarize you with different types of trigonometric equations and show you some techniques you can use to solve them.

One of the easiest things to do is determining if you're looking for a general solution or a solution set over a specific (given) interval. This will determine what your solution will look like when you're finished solving the problem.

Once you know the form your solution should take, focus on solving the equation. **Remember that you've solved equations before**. Solving trigonometric equations doesn't usually require much additional work. For example, if you can solve the algebraic equation $2(1-x^2)-x=1$, you can solve the trigonometric equation $2\sin^2 x-\cos x=1$ (Hint: *First use an identity*).

Steps for solving $2\left(1-x^2\right)-x=1$	Steps for solving $2\sin^2 x-\cos x=1$
$\left.\begin{aligned} &2\left(1-x^2\right)-x=1 \\ &2-2x^2-x=1 \\ &0=2x^2+x-1 \\ &0=(2x-1)(x+1) \\ &2x-1=0 \quad \text{or} \quad x+1=0 \\ &\quad x=\frac{1}{2} \quad \text{or} \quad x=-1 \end{aligned}\right\}$	$2\sin^2 x-\cos x=1$ (*use the identity* $\sin^2 x=1-\cos^2 x$) $\left\{\begin{aligned} &2\left(1-\cos^2 x\right)-\cos x=1 \\ &2-2\cos^2 x-\cos x=1 \\ &0=2\cos^2 x+\cos x-1 \\ &0=(2\cos x-1)(\cos x+1) \\ &2\cos x-1=0 \quad \text{or} \quad \cos x+1=0 \\ &\cos x=\frac{1}{2} \quad \text{or} \quad \cos x=-1 \end{aligned}\right.$ $x=\frac{\pi}{3}+2\pi k \qquad x=\pi+2\pi k$ $x=\frac{5\pi}{3}+2\pi k$

The steps taken in the braced off sections of both equations are identical. Due to the additional element of the trigonometric function the equation on the right may look much more complicated, but after using the trigonometric identity, there in only one additional step necessary to solve for the general solution.

THINGS TO KEEP IN MIND WHEN ATTEMPTING TO SOLVE A TRIGONOMETRIC EQUATION:

1. Determine if you are looking for a general solution or a solution over a specific interval.

2. Regardless of whether or not you are looking for a general solution or a solution over a specific interval, as soon as you eliminate the trigonometric function from your work, be certain to include the $+2\pi k$ (for equations using sine, cosine, cosecant, or secant) or $+\pi k$ (for equations using tangent or cotangent).

3. Check your answers for extraneous solutions (based on the original statement of the problem). This can happen in any equation containing a tangent, cotangent, secant, or cosecant in the original problem statement.

4. It is possible that your answers might not be conventional "nice" angles like $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, etc. There may be an inverse trig function involved in your answer, ex. $\sin^{-1}\frac{5}{13}$ or $\tan^{-1}\frac{24}{7}$.

5. Try using algebraic techniques to help you solve an equation before using any trigonometric manipulation. Factoring is incredibly useful in solving equations.

6. If algebra doesn't work to help you solve the equation, try getting the equation into terms of a single trig function. The Pythagorean Identities are often helpful for doing this.

7. Try to avoid using double angle, half angle, or other complicated trig formulas when possible. Those techniques should only be used when absolutely necessary, i.e. when all of the other techniques have been tried and didn't work.

Solve each of the following trigonometric equations, providing both a) the general solution, and b) the solution set over the interval $[0, 2\pi)$.

1) $\cot\left(x - \frac{\pi}{2}\right) + 1 = 0$

2) $2\cos 3\theta + 1 = 0$

Solve each of the following trigonometric equations, providing both a) the general solution, and b) the solution set over the interval $[0, 2\pi)$.

3) $\sin 2\varphi - \cos\varphi = 0$

4) $2\tan 2\alpha = \sec 2\alpha$