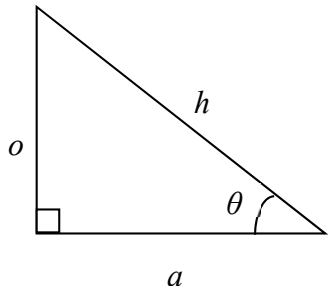
MSLC Trigonometry Handout

Trigonometric Functions:

$\sin\theta = \frac{o}{h}$ $\quad \csc\theta = \frac{h}{o}$

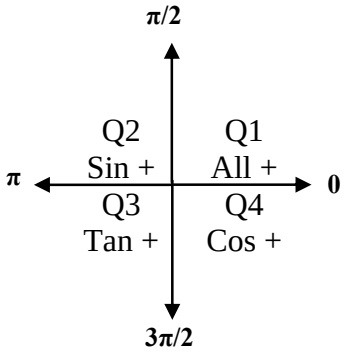
$\cos\theta = \frac{a}{h}$ $\quad \sec\theta = \frac{h}{a}$

$\tan\theta = \frac{o}{a}$ $\quad \cot\theta = \frac{a}{o}$



Trig Values for Special Angles:

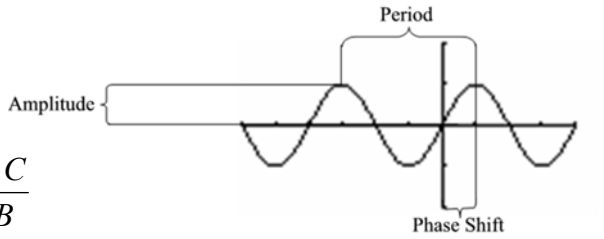
θ	**0** $(0°)$	**π/6** $(30°)$	**π/4** $(45°)$	**π/3** $(60°)$	**π/2** $(90°)$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}=1$
$\cos\theta$	$1=\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined



Graphing Trigonometric Functions:

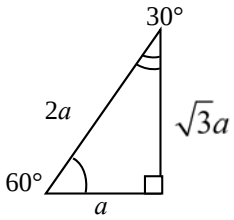
$y = A\sin(Bx + C)$ $\quad y = A\cos(Bx + C)$

$Amplitude = |A|$ $\quad Period = \frac{2\pi}{B}$ $\quad PhaseShift = \frac{-C}{B}$

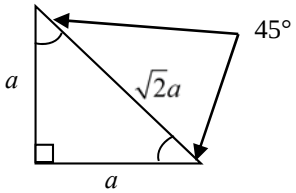


Special Triangles:

30 – 60 - 90 triangle:



45 – 45- 90 triangle:



Law of Sines:

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

Law of Cosines:

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Pythagorean Theorem:

$a^2 + b^2 = c^2$

Ratio Identities:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Co-function Identities:

$$\sin\theta = \cos\left(\pi/2 - \theta\right) \qquad \tan\theta = \cot\left(\pi/2 - \theta\right) \qquad \sec\theta = \csc\left(\pi/2 - \theta\right)$$

$$\cos\theta = \sin\left(\pi/2 - \theta\right) \qquad \cot\theta = \tan\left(\pi/2 - \theta\right) \qquad \csc\theta = \sec\left(\pi/2 - \theta\right)$$

Even/Odd Identities:

$$\sin\left(-\theta\right) = -\sin\theta$$

$$\cos\left(-\theta\right) = \cos\theta$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Addition & Subtraction Identities:

$$\sin\left(x+y\right) = \left(\sin x\right)\left(\cos y\right) + \left(\sin y\right)\left(\cos x\right) \qquad \cos\left(x+y\right) = \left(\cos x\right)\left(\cos y\right) - \left(\sin x\right)\left(\sin y\right) \qquad \tan\left(x+y\right) = \frac{\tan x + \tan y}{1 - \left(\tan x\right)\left(\tan y\right)}$$

$$\sin\left(x-y\right) = \left(\sin x\right)\left(\cos y\right) - \left(\sin y\right)\left(\cos x\right) \qquad \cos\left(x-y\right) = \left(\cos x\right)\left(\cos y\right) + \left(\sin x\right)\left(\sin y\right) \qquad \tan\left(x-y\right) = \frac{\tan x - \tan y}{1 + \left(\tan x\right)\left(\tan y\right)}$$

Double-Angle Identities:

$$\sin\left(2x\right) = 2\left(\sin x\right)\left(\cos x\right)$$

$$\cos\left(2x\right) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan\left(2x\right) = \frac{2\tan x}{1 - \tan^2 x}$$

Power Reducing Formulas:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Product-to-Sum Identities:

$$\sin x\cos y = \frac{1}{2}\left[\sin\left(x+y\right) + \sin\left(x-y\right)\right]$$

$$\cos x\sin y = \frac{1}{2}\left[\sin\left(x+y\right) - \sin\left(x-y\right)\right]$$

$$\sin x\sin y = \frac{1}{2}\left[\cos\left(x-y\right) - \cos\left(x+y\right)\right]$$

$$\cos x\cos y = \frac{1}{2}\left[\cos\left(x+y\right) + \cos\left(x-y\right)\right]$$

Sum-to-Product Identities:

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

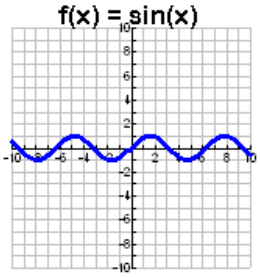
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

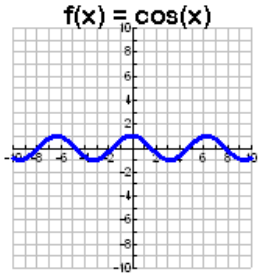
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

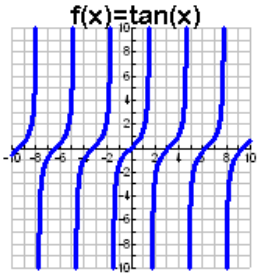
Half-Angle Identities:

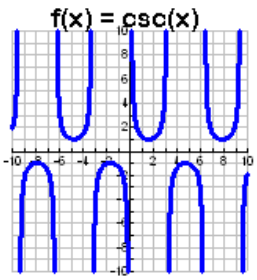
$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} \qquad \sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \qquad \tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$$

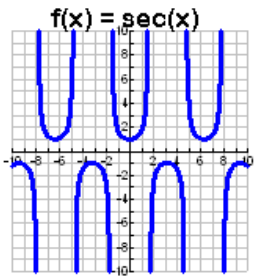
Trig Graphs:

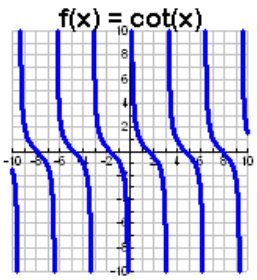










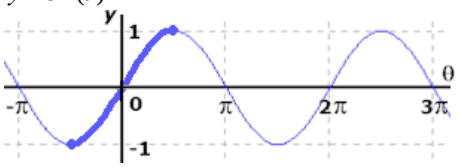


Inverse Trig functions:

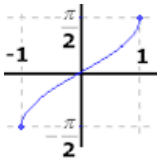
$y = \arcsin x \ \ (= \sin^{-1} x)$

DOMAIN: **RANGE:**

$y = \sin(\theta)$



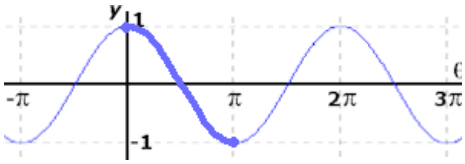
$y = \sin^{-1}(x)$



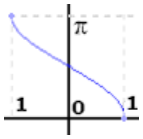
$-1 \le x \le 1$ $\quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = \arccos x \ \ (= \cos^{-1} x)$

$y = \cos(\theta)$



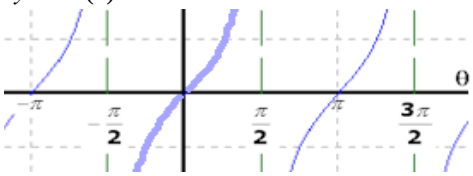
$y = \cos^{-1}(x)$



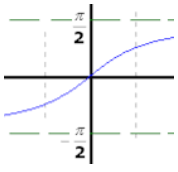
$-1 \le x \le 1$ $\quad 0 \le y \le \pi$

$y = \arctan x \ \ (= \tan^{-1} x)$

$y = \tan(\theta)$



$y = \tan^{-1}(x)$



$-\infty < x < \infty$ $\quad -\frac{\pi}{2} < y < \frac{\pi}{2}$

Radians & Degrees

Converting Radians to Degrees: $\theta \cdot \frac{180}{\pi}$

Converting Degrees to Radians: $\theta \cdot \frac{\pi}{180}$

The Unit Circle